

Solutions to Maths workbook – 1 | Trigonometric Identities and Equations

Miscellaneous Question Bank

Level – 1 & 2

1.(D)

Function	Period
$\sin \frac{\pi}{2} x$	4
$\cos \frac{\pi}{3} x$	6
$\tan \frac{\pi}{3} x$	3

Take LCM of (4, 6, 3) = 12

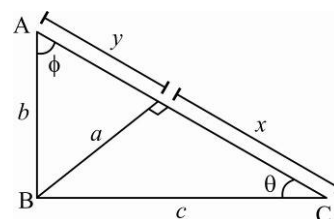
2.(B) $2\sqrt{2}a = H$

$$\theta + \phi = \frac{\pi}{2} \Rightarrow 1 = \tan \theta \tan \phi$$

$$\cot \theta = \frac{x}{a} \quad \text{and} \quad \cot \phi = \frac{y}{a}$$

$$\cot \theta + \cot \phi = \frac{x+y}{a} = \frac{H}{a} = 2\sqrt{2} \Rightarrow \frac{1}{\tan \theta} + \tan \theta = 2\sqrt{2}$$

$$\tan^2 \theta - 2\sqrt{2} \tan \theta + 1 = 0 \Rightarrow \tan \theta = \frac{2\sqrt{2} \pm 2}{2} = \sqrt{2} \pm 1 \Rightarrow \theta = \frac{45^\circ}{2}, \frac{135^\circ}{2}$$



3.(B) $AD = DB \Rightarrow \angle ADE = \angle BDE \Rightarrow ED = ED$ (Common to both triangles)

Hence both Δ 's are concurrent. Hence $AE = BE$

$$\Rightarrow \beta = \gamma \Rightarrow AC = 12, AB = 20, BC = 16$$

$$\Rightarrow \tan \gamma = \frac{AC}{BC} = \frac{12}{16} = \frac{3}{4}; \quad \alpha + \beta = 90 - \gamma; \quad \alpha + \gamma = 90 - \gamma; \quad \tan \alpha = \frac{CE}{AC}$$

$$CE = AC \tan(90 - 2\gamma) = 12 \times \frac{(1 - \tan^2 \gamma)}{2 \tan \gamma} = 12 \times \frac{\left(1 - \frac{9}{16}\right)}{2 \times \frac{3}{4}} = 8 \times \frac{7}{16} = \frac{7}{2}$$

$$\text{Area} = \frac{1}{2} \times AC \times CE = \frac{1}{2} \times 12 \times \frac{7}{2} = 21$$

$$4.(A) \delta = \gamma + \beta \quad \text{and} \quad \delta + \beta = 3\gamma \Rightarrow \tan(3\gamma) = \frac{3 \tan \gamma - \tan^3 \gamma}{1 - 3 \tan^2 \gamma} = -\frac{117}{44}$$

$$5.(B) \cos(\alpha + \beta) = \frac{AC}{AB} = \frac{12}{20} = \frac{3}{5}$$

$$6.(C) \quad P = \cos \frac{\pi}{20} \cos \frac{3\pi}{20} \cos \frac{7\pi}{20} \cos \frac{9\pi}{20} \Rightarrow P = \cos \frac{\pi}{20} \sin \frac{\pi}{20} \cos \frac{3\pi}{20} \sin \frac{3\pi}{20}$$

$$P = \frac{1}{4} \sin \frac{\pi}{10} \sin \frac{3\pi}{10} = \frac{1}{4} \frac{\sqrt{5}-1}{4} \times \frac{\sqrt{5}+1}{4} = \frac{1}{16}$$

$$Q = \cos \frac{\pi}{11} \cos \frac{2\pi}{11} \cos \frac{4\pi}{11} \cos \frac{8\pi}{11} \cos \frac{16\pi}{11} \Rightarrow Q = \frac{1}{2^5} \frac{\sin 2^5 \frac{\pi}{11}}{\sin \frac{\pi}{11}} = \frac{1}{32} \frac{\sin \left(3\pi - \frac{\pi}{11} \right)}{\sin \frac{\pi}{11}} = \frac{1}{32}$$

$$\Rightarrow \frac{P}{Q} = \frac{\frac{1}{16}}{\frac{1}{32}} = 2$$

$$7.(B) \quad \sin(2A+B) = \frac{1}{2} \Rightarrow 2A+B = 30^\circ \text{ or } 150^\circ$$

$$\begin{array}{lll} A, B, C \text{ are in AP} & \Rightarrow & B = 60^\circ \quad \therefore \quad 2A = -30^\circ \text{ or } 90^\circ \\ \Rightarrow 2A = 90^\circ & \Rightarrow & A = 45^\circ \quad \therefore \quad C = 180^\circ - A - B = 75^\circ \end{array}$$

$$8.(A) \quad \cos \theta + \sin \theta = \frac{1}{5}, \quad 0 < \theta < \pi; \quad \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}} + \frac{2 \tan \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}} = \frac{1}{5}$$

$$\Rightarrow 5 - 5 \tan^2 \frac{\theta}{2} + 10 \tan \frac{\theta}{2} = 1 + \tan^2 \frac{\theta}{2} \Rightarrow 6 \tan^2 \frac{\theta}{2} - 10 \tan \frac{\theta}{2} - 4 = 0$$

$$\Rightarrow 6 \tan^2 \frac{\theta}{2} - 12 \tan \frac{\theta}{2} + 2 \tan \frac{\theta}{2} - 4 = 0 \Rightarrow \tan \frac{\theta}{2} = 2, \tan \frac{\theta}{2} = -\frac{1}{3}$$

$$\theta \in (0, \pi)$$

$$\text{So } \frac{\theta}{2} \in \left(0, \frac{\pi}{2} \right). \text{ Hence } \tan \frac{\theta}{2} = -\frac{1}{3} \text{ is discarded } \tan \frac{\theta}{2} = 2$$

$$\Rightarrow \tan \theta = \frac{2 \tan(\theta/2)}{1 - \tan^2(\theta/2)} = \frac{4}{1-4} = -\frac{4}{3}$$

$$9.(AD) \quad \sin \theta + \sin \phi = a \quad \dots .(i) \quad \& \quad \cos \theta + \cos \phi = b \quad \dots .(ii)$$

Squaring and adding

$$\sin^2 \theta + \sin^2 \phi + 2 \sin \theta \sin \phi = a^2 \Rightarrow \cos^2 \theta + \cos^2 \phi + 2 \cos \theta \cos \phi = b^2$$

$$2 + 2(\cos \theta \cos \phi + \sin \phi \sin \theta) = a^2 + b^2 \Rightarrow \cos(\theta - \phi) = \frac{a^2 + b^2 - 2}{2}$$

$$2 \cos^2 \left(\frac{\theta - \phi}{2} \right) - 1 = \frac{a^2 + b^2}{2} - 1 \Rightarrow \cos^2 \left(\frac{\theta - \phi}{2} \right) = \frac{a^2 + b^2}{4}$$

$$\cos \left(\frac{\theta - \phi}{2} \right) = \pm \frac{1}{2} \sqrt{a^2 + b^2}$$

10.(BD) $0 < x < \pi$

$$0 < y < \pi$$

$$-\pi < -y < 0$$

$$-\pi < x - y < \pi$$

$$0 < x + y < 2\pi$$

$$\sin(x - y) = \frac{1}{2}$$

$$x - y = \frac{\pi}{6} \text{ and } x - y = \frac{5\pi}{6}$$

$$x + y = \frac{\pi}{3} \text{ and } x + y = \frac{5\pi}{3}$$

$$x = \frac{\pi}{4} \quad y = \frac{\pi}{12}$$

$$x = \frac{11\pi}{12} \quad y = \frac{3\pi}{4}$$

11.(C) Given $S_n = \cos^n \theta + \sin^n \theta$

$$\begin{aligned} \therefore 3S_4 - 2S_6 &= 3[(\cos^4 \theta + \sin^4 \theta)] - 2[\cos^6 \theta + \sin^6 \theta] = 3[(\cos^2 \theta + \sin^2 \theta)^2 - 2\sin^2 \theta \cos^2 \theta] \\ &\quad - 2[(\cos^2 \theta + \sin^2 \theta)(\cos^4 \theta + \sin^4 \theta - \cos^2 \theta \sin^2 \theta)] \\ &= 3[1 - 2\sin^2 \theta \cos^2 \theta] - 2(\cos^2 \theta + \sin^2 \theta)[(\cos^2 \theta + \sin^2 \theta)^2 - 3\cos^2 \theta \sin^2 \theta] \\ &= 3 - 6\sin^2 \theta \cos^2 \theta - 2 + 6\cos^2 \theta \sin^2 \theta = 1 \end{aligned}$$

12.(D) Given, $x \sin^3 \theta + y \cos^3 \theta = \sin \theta \cos \theta$

$$\Rightarrow y \cos \theta \sin^2 \theta + y \cos^3 \theta = \sin \theta \cos \theta \quad [\because x \sin \theta = y \cos \theta]$$

$$\Rightarrow y \cos \theta (\sin^2 \theta + \cos^2 \theta) = \sin \theta \cos \theta \Rightarrow y = \sin \theta$$

$$\text{Now, } x \sin \theta = \sin \theta \cos \theta \Rightarrow x = \cos \theta \quad \therefore x^2 + y^2 = \cos^2 \theta + \sin^2 \theta = 1$$

13.(D) $\cos x + \cos^2 x = 1 \Rightarrow \cos x = \sin^2 x$

$$\begin{aligned} \text{Now, } \sin^{12} x + 3\sin^{10} x + 3\sin^8 x + \sin^6 x - 1 \\ = \cos^6 x + 3\cos^5 x + 3\cos^4 x + \cos^3 x - 1 = (\cos^2 x + \cos x)^3 - 1 = 1 - 1 = 0 \end{aligned}$$

14.(D) Given, $f(x) = \sin^4 x + \cos^4 x = (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x$

$$\Rightarrow f(x) = 1 - \frac{1}{2} \sin^2 2x. \quad \text{Also, } 0 \leq \sin^2 2x \leq 1 \quad \therefore \text{ Minimum value of } f(x) \text{ is } 1 - \frac{1}{2} = \frac{1}{2}$$

15.(C) Given, $4\sin^4 x + \cos^4 x - 1 = 0$

$$\Rightarrow 4\sin^4 x + (\cos^2 x - 1)(\cos^2 x + 1) = 0 \Rightarrow 4\sin^4 x - \sin^2 x(1 - \sin^2 x + 1) = 0$$

$$\Rightarrow \sin^2 x(5\sin^2 x - 2) = 0 \Rightarrow \sin x = 0 \text{ or } \pm\sqrt{\frac{2}{5}}. \text{ Hence, } x = n\pi \text{ is the required solution.}$$

16.(C) ABCD is a cyclic quadrilateral $\Rightarrow A + C = 180^\circ, B + D = 180^\circ \Rightarrow C = 180^\circ - A, D = 180^\circ - B$
 $\Rightarrow \cos C = \cos(180^\circ - A) = -\cos A, \cos D = \cos(180^\circ - B) = -\cos B$
 $\Rightarrow \cos A + \cos C = 0, \cos B + \cos D = 0 \Rightarrow \cos A + \cos B + \cos C + \cos D = 0$

17.(C) $2P_6 - 3P_4 + 1 = 2(1 - 3\sin^2 x \cos^2 x) - 3\left[(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x\right] + 1$
 $= 2 - 6\sin^2 x \cos^2 x - 3(1 - 2\sin^2 x \cos^2 x) + 1 = 0$

18.(A) Given that $\tan \theta_1 \cdot \tan \theta_2 = k$
 $\frac{\cos(\theta_1 - \theta_2)}{\cos(\theta_1 + \theta_2)} = \frac{\cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2}{\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2} = \frac{1 + \tan \theta_1 \tan \theta_2}{1 - \tan \theta_1 \tan \theta_2} = \frac{1 + k}{1 - k}$

19.(D) $3\sin 2x + 4\cos 2x + 3 \in [-2, 8]$

20.(D) Given expression $= \frac{(\cos A + \cos 5A) + (\cos 3A + \cos 7A)}{(\sin A + \sin 5A) + (\sin 3A + \sin 7A)} = \frac{2\cos 3A \cos 2A + 2\cos 5A \cos 2A}{2\sin 3A \cos 2A + 2\sin 5A \cos 2A}$
 $= \frac{2\cos 2A(\cos 3A + \cos 5A)}{2\cos 2A(\sin 3A + \sin 5A)} = \frac{2\cos 4A \cos A}{2\sin 4A \cos A} = \cot 4A$

21.(D) $(\sin \theta + \sec \theta)^2 + (\cos \theta + \sec \theta)^2 = 7 + \tan^2 \theta + \cot^2 \theta \quad \therefore \text{Minimum value} = 9$

22.(C) $2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \frac{3}{2} \Rightarrow 2\cos\left(\frac{\pi}{3}\right)\cos\left(\frac{x-y}{2}\right) = \frac{3}{2} \Rightarrow 2 \cdot \frac{1}{2} \cdot \cos\left(\frac{x-y}{2}\right) = \frac{3}{2}$
 $\Rightarrow \cos\left(\frac{x-y}{2}\right) = \frac{3}{2} > 1 \quad \therefore \text{Given equation has no solution.}$

23.(C) $\log_{\cos x} \sin x + \log_{\sin x} \cos x = 2 \Rightarrow \log_{\cos x} \sin x = 1 \Rightarrow \sin x = \cos x \Rightarrow x = \frac{\pi}{4}$

24.(B) One root lies in the interval $\left(-\frac{\pi}{2}, 0\right)$

25.(B) $\cos 6\theta + \cos 4\theta + \cos 2\theta + 1 = 0 \Rightarrow 2\cos 5\theta \cdot \cos \theta + 2\cos^2 \theta = 0 \Rightarrow 2\cos \theta (\cos 5\theta + \cos \theta) = 0$
 $\Rightarrow 4\cos \theta \cdot \cos 3\theta \cdot \cos 2\theta = 0 \Rightarrow \cos \theta = 0 \text{ or } \cos 2\theta = 0 \text{ or } \cos 3\theta = 0$
 $\Rightarrow \theta = (2n+1)\frac{\pi}{2} \text{ or } (2n+1)\frac{\pi}{4} \text{ or } (2n+1)\frac{\pi}{6} \Rightarrow \theta = \frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{6}, \frac{3\pi}{4}, \frac{5\pi}{6}$

26.(AD) Use $\sec^2 \theta - \tan^2 \theta = 1$

27.(C) Expanding D, we get $D = 1 + \sin \theta \cos \theta - \cos \theta(-\sin \theta - \cos \theta) + (-\sin^2 \theta + 1)$
 $= 2 + 2\sin \theta \cos \theta + \cos 2\theta = 2 + \sin 2\theta + \cos 2\theta = 2 + \sqrt{2} \cos\left(2\theta - \frac{\pi}{4}\right)$
 As $-1 \leq \cos\left(2\theta - \frac{\pi}{4}\right) \leq 1, 2 - \sqrt{2} \leq D \leq 2 + \sqrt{2}$

$$28.(B) \tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} \Rightarrow \sqrt{3} = \tan 3 \times \frac{\pi}{9} = \frac{3 \tan \frac{\pi}{9} - \tan^3 \frac{\pi}{9}}{1 - 3 \tan^2 \frac{\pi}{9}}$$

$$\Rightarrow \left[\sqrt{3} \left(1 - 3 \tan^2 \frac{\pi}{9} \right) \right]^2 = \left(3 \tan \frac{\pi}{9} - \tan^3 \frac{\pi}{9} \right)^2 \Rightarrow \tan^6 \frac{\pi}{9} - 33 \tan^4 \frac{\pi}{9} + 27 \tan^2 \frac{\pi}{9} = 3 = \tan^2 \frac{\pi}{3}$$

$$29.(D) \frac{a}{b} = \frac{\sin \alpha + \sin \beta}{\cos \alpha + \cos \beta} = \frac{2 \sin \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)}{2 \cos \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)} = \tan \left(\frac{\alpha + \beta}{2} \right)$$

$$\therefore \sin(\alpha + \beta) = \frac{2 \cdot \left(\frac{a}{b} \right)}{1 + \left(\frac{a^2}{b^2} \right)} = \frac{2ab}{a^2 + b^2}$$

$$30.(C) \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \cos \frac{14\pi}{15} = -\cos \left(\frac{\pi}{15} \right) \cos \left(\frac{2\pi}{15} \right) \cos \left(\frac{4\pi}{15} \right) \cos \left(\frac{8\pi}{15} \right) = \frac{1}{16}$$

31.(A) Sum of the roots

$$= \sin^2 18^\circ + \cos^2 36^\circ = \left[\frac{\sqrt{5}-1}{4} \right]^2 + \left[\frac{\sqrt{5}+1}{4} \right]^2 = \frac{1}{16} \left[(\sqrt{5}+1)^2 + (\sqrt{5}-1)^2 \right] = \left(\frac{1}{16} \right) [2(5+1)] = \frac{3}{4}$$

$$\text{Product of the roots} = \sin^2 18^\circ \cos^2 36^\circ = \left[\frac{\sqrt{5}-1}{4} \right]^2 \left[\frac{\sqrt{5}+1}{4} \right]^2 = \left[\frac{5-1}{4 \cdot 4} \right]^2 = \frac{1}{16}$$

$$\Rightarrow \text{Required quadratic equation is } x^2 - \frac{3x}{4} + \frac{1}{16} = 0 \quad \text{or} \quad 16x^2 - 12x + 1 = 0$$

$$32.(B) \tan \theta + \tan 4\theta + \tan 7\theta = \tan \theta \cdot \tan 4\theta \cdot \tan 7\theta \Rightarrow \tan \theta + \tan 4\theta = -\tan 7\theta + \tan \theta \cdot \tan 4\theta \cdot \tan 7\theta$$

$$\Rightarrow \tan \theta + \tan 4\theta = -\tan 7\theta (1 - \tan \theta \cdot \tan 4\theta) \Rightarrow \frac{\tan \theta + \tan 4\theta}{1 - \tan \theta \cdot \tan 4\theta} = -\tan 7\theta$$

$$\Rightarrow \tan(\theta + 4\theta) = -\tan 7\theta \Rightarrow \tan 5\theta = \tan(-7\theta) \Rightarrow 5\theta = n\pi + (-7\theta)$$

$$\Rightarrow 12\theta = n\pi \Rightarrow \theta = \frac{n\pi}{12} \quad (\text{where } n \in \mathbb{I})$$

$$33.(C) \sin(x+y) = 2 \sin x \sin y$$

$$\sin(x+y) = \cos(x-y) - \cos(x+y)$$

$$\Rightarrow \sin \left(x + y + \frac{\pi}{4} \right) = \sin \frac{\pi}{6} \text{ or } \sin \frac{5\pi}{6}$$

$$x + y + \frac{\pi}{4} = \frac{\pi}{6} \text{ or } \frac{5\pi}{6} \text{ and } x - y = \frac{\pi}{4}$$

$$x = \frac{\pi}{12}, \frac{5\pi}{12} \text{ for } x = \frac{\pi}{12}, y < 0 \Rightarrow x = \frac{5\pi}{12} \Rightarrow \lambda = 12$$

34.(C) $8x^3 - 4x^2 - 4x + 1 = 0$

$$x \rightarrow -x \Rightarrow x^3 + 4x^2 - 4x - 8 = 0$$

35.(D) On adding and subtracting

$$x = \frac{3 - \cos 4\theta + 4 \sin 2\theta}{2}; \quad y = \frac{3 - \cos 4\theta - 4 \sin 2\theta}{2}$$

$$x = \frac{3 - (1 - 2 \sin^2 2\theta) + 4 \sin 2\theta}{2} \quad y = \frac{3 - (1 - 2 \sin^2 2\theta) - 4 \sin 2\theta}{2}$$

$$x = 1 + 2 \sin 2\theta + \sin^2 2\theta; \quad y = 1 - 2 \sin 2\theta + \sin^2 2\theta$$

$$x = (1 + \sin 2\theta)^2; \quad y = (1 + \sin 2\theta)^2 \Rightarrow \sqrt{x} + \sqrt{y} = 2$$

36.(B) $(a+1)^2 + \operatorname{cosec}^2\left(\frac{\pi a}{2} + \frac{\pi x}{2}\right) - 1 = 0$ or $(a+1)^2 + \cot^2\left(\frac{\pi a}{2} + \frac{\pi x}{2}\right) = 0$

from option [B] If $a = -1 \Rightarrow \tan^2 \pi x / 2 = 0 \Rightarrow x/2 \in I$

37.(B) $A^2 + B^2 = 3 + 2\left[\cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7}\right] = 3 + 2\left(-\frac{1}{2}\right) = 2 \Rightarrow \sqrt{A^2 + B^2} = \sqrt{2}$

38.(C) $k = \frac{2 \tan x}{1 + \tan^2 x} = \sin 2x \Rightarrow \sin 2C = \sin 2B$

But $\angle C > \angle B$

$$2C = \pi - 2B \Rightarrow B + C = \frac{\pi}{2} \therefore \angle A = \frac{\pi}{2}$$

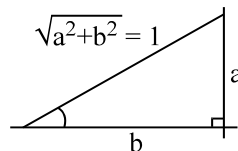
39.(A) $b^2 \cos^2 x = (1 - a \sin x)^2$

$$b^2(1 - \sin^2 x) = 1 + a^2 \sin^2 x - 2a \sin x$$

$$(a^2 + b^2) \sin^2 x - 2a \sin x + 1 - b^2 = 0$$

$$\sin^2 x - 2a \sin x + a^2 = 0$$

$$(a - \sin x)^2 = 0; \sin x = a \Rightarrow \tan x = a/b$$



40.(A) $-3 + 4 \cos^2 x = \frac{1}{3} \Rightarrow \cos^2 x = \frac{5}{6} \Rightarrow \sin^2 x = \frac{1}{6}$

$$\frac{\sin 3x}{\sin x} = 3 - 4 \sin^2 x = \frac{7}{3}$$

41.(D) On rationalizing; we get $\frac{1 - \sin x + 1 + \sin x + 2 |\cos x|}{1 - \sin x - 1 + \sin x} = \frac{2(1 + |\cos x|)}{-2(\sin x)} = \frac{1 - \cos x}{-(\sin x)}$

42.(A) Let $f(x) = (\sqrt{\sin \alpha} x + b)^2$ now compare the coefficient and eliminate b . divide by $\cos^2 \alpha$ to get $(\tan x - 1)$

$$(\tan x + 2) = 0 \Rightarrow \alpha = \frac{\pi}{4} \text{ or } \pi - \tan^{-1}(2)$$

43.(D) $\cot x + \frac{\cos(60+x)}{\sin(60+x)} + \frac{\cos(x-60)}{\sin(x-60)} = \frac{\cos x}{\sin x} + \frac{\sin(2x)}{\sin(x+60)\sin(x-60)} = \frac{\cos x}{\sin x} + \frac{8 \sin x \cos x}{4 \sin^2 x - 3}$

$$= \frac{4 \sin^2 x \cos x - 3 \cos x + 8 \sin^2 x \cos x}{4 \sin^3 x - 3 \sin x} = \frac{3[3 \cos x - 4 \cos^3 x]}{\sin^3 x} = 3 \cot 3x \quad \frac{3[1 - 3 \tan^2 x]}{3 \tan x - \tan^3 x}$$

44.(D) Let $\cos 2\theta = t$
 $\therefore 8t + \frac{8}{t} = 65 \Rightarrow 8t^2 - 65t + 8 = 0 \Rightarrow 8t^2 - 64t - t + 8 \Rightarrow 8t(t-8) - (t-8) = 0$
 $t = 8$ or $t = \frac{1}{8}$ ($t = 8$ is rejected)
 $\therefore \cos 2\theta = \frac{1}{8}; \quad 2\cos^2 \theta - 1 = \frac{1}{8} \Rightarrow \cos^2 \theta = \frac{9}{16} \Rightarrow \cos \theta = \frac{3}{4}$

45.(A) In 2nd quadrant $\sin x < \cos x$ is False
 In 4th quadrant $\cos x < \tan x$ is False
 in 3rd quadrant, i.e. $\left(\frac{5\pi}{4}, \frac{3\pi}{2}\right)$ if $\tan x < \cot x \Rightarrow \tan^2 x < 1$ which is not correct
 Hence A can be correct

now $\sin x < \cos x$ is true in $\left(0, \frac{\pi}{4}\right)$ and $\tan x < \cot x$ is also true
 $\therefore$ only the value of x for which $\cos x < \tan x$ is to be determined
 $\therefore$ now $\cos x = \tan x$
 i.e. $\cos^2 x = \sin x$ or $1 - \sin^2 x = \sin x \Rightarrow \sin^2 x + \sin x - 1 > 0$
 $\sin x = \frac{-1 \pm \sqrt{5}}{2}; \quad \sin x = \frac{\sqrt{5}-1}{2} \Rightarrow x = \sin^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$
 $\therefore \cos x < \tan x$ in $\left(\sin^{-1}\frac{\sqrt{5}-1}{2}, \frac{\pi}{4}\right)$ and $\cos x > \tan x$ in $\left(0, \sin^{-1}\frac{\sqrt{5}-1}{4}\right)$

46.(A) $a \sin x + c < 0$
 $\sin x < -\frac{c}{a}; \quad -\frac{c}{a} > \sin x; \quad -\frac{c}{a} > 1; \quad -c > a \Rightarrow a + c < 0$

47.(A) Let $\tan x = a; \tan y = b$
 $\Rightarrow a + b = 25 \dots(1)$
 and $\frac{1}{a} + \frac{1}{b} = 30$
 $\Rightarrow \frac{a+b}{ab} = 30 \Rightarrow ab = \frac{25}{30} = \frac{5}{6} \therefore \tan(x+y) = \frac{a+b}{1-ab} = \frac{25}{1-(5/6)} = 25 \times 6 = 150$

48.(B) $p^2 \sec^2 \theta + p^2 \operatorname{cosec}^2 \theta = (2\sqrt{2})^2 p^2$

$\Rightarrow \frac{1}{\sin^2 \theta \cos^2 \theta} = 8$

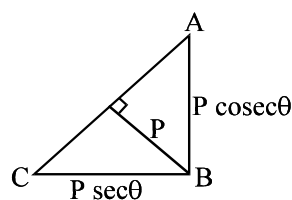
$\sin^2 2\theta = 1/2 = \left(\frac{1}{\sqrt{2}}\right)^2$

$2\theta = n\pi + \pi/4$

$\theta = n\pi/2 + \pi/8$

for $n = 0 \Rightarrow \theta = \pi/8$

for $n = 1 \Rightarrow \theta = 3\pi/8$



49.(C) We know that:
 $\cos a + \cos(a+d) + \dots \cos(a+(n-1)d)$

$$= \frac{\sin\left(\frac{nd}{2}\right) \cos\left(a + \left(\frac{n-1}{2}\right)d\right)}{\sin\left(\frac{d}{2}\right)} \quad \therefore \quad \frac{\sin 6\theta \cos 5\theta}{\sin \theta} = \frac{\cos m\theta \cdot \sin n\theta}{\sin \theta} \Rightarrow n = 6 \text{ and } m = 5$$

50.(A) $\cos 36^\circ + \cos 84^\circ + \cos 156^\circ$
 $\cos(60^\circ - 24^\circ) + \cos(60^\circ + 24^\circ) - \cos 24^\circ$
 $= 2 \cos 60^\circ \cos 24^\circ - \cos 24^\circ = \cos 24^\circ - \cos 24^\circ = 0$

51.(D) $\sin x + \sin y = a$ and $\cos x + \cos y = 2a$

On squaring and adding these equations we get $2 + 2\cos(x-y) = 5a^2 \Rightarrow \cos(x-y) = \frac{5a^2 - 2}{2}$

52.(A) $P(x) = \sqrt{3 + 2(\cos x + \cos x + \cos 2x)} = \sqrt{3 + 2(2\cos x + 2\cos^2 x - 1)} = \sqrt{4\cos^2 x + 4\cos^2 x + 1} = |2\cos x + 1|$

53.(A) Let $\frac{(\cos \theta)}{a} = \frac{(\sin \theta)}{b} = k$, so that $\cos \theta = ak$ and $\sin \theta = bk$.

Then, $a \cos 2\theta + b \sin 2\theta = a(1 - 2\sin^2 \theta) + 2b \sin \theta \cos \theta = a - 2ab^2k^2 + 2b \cdot bk \cdot ak = a - 2ab^2k^2 + 2ab^2k^2 = a$

54.(D) $f(\theta) = |\sin \theta| + |\cos \theta|, \forall \theta \in R$ Clearly, $f(\theta) > 0$.

Also, $f^2(\theta) = \sin^2 \theta + \cos^2 \theta + |2 \sin \theta \cdot \cos \theta| = 1 + |\sin 2\theta|$

We know that $0 \leq |\sin 2\theta| \leq 1 \Rightarrow 1 \leq f^2(\theta) \leq 2 \Rightarrow 1 \leq f(\theta) \leq \sqrt{2}$

55.(B) If $a, b > 0$

Using A.M. $\geq$ G.M. we get $\frac{1}{a} + \frac{1}{b} \geq \frac{2}{ab}$

$$\Rightarrow f(x) \geq \frac{2}{\sqrt{\cos\left(\frac{\pi}{6} - x\right) \cos\left(\frac{\pi}{6} - x\right)}} = \frac{2}{\sqrt{\cos^2 \frac{\pi}{6} - \sin^2 x}} = \frac{2}{\sqrt{\frac{3}{4} - \frac{1 - \cos 2x}{2}}} = \frac{2}{\sqrt{\frac{1}{4} + \frac{\cos 2x}{2}}}$$

Now for $0 \leq x \leq \frac{\pi}{3}, \frac{-1}{2} \leq \cos 2x \leq 1$

$$\Rightarrow 0 \leq \sqrt{\frac{1}{4} + \frac{\cos 2x}{2}} \leq \frac{\sqrt{3}}{2} \Rightarrow f(x) \geq \frac{4}{\sqrt{3}}$$

Since 'f' is continuous range of 'f' is $\left[\frac{4}{\sqrt{3}}, \infty\right)$.